

## The IMF of simple and composite populations

Pavel Kroupa

*Argelander-Institut für Astronomie, Universität Bonn, Germany*

**Abstract.** The combination of a finite time-scale for star formation, rapid early stellar evolution and rapid stellar-dynamical processes imply that the stellar IMF cannot be inferred for *any* star cluster independently of its age (the CLUSTER IMF THEOREM). The IMF can nevertheless be constrained statistically by evolving many theoretical populations drawn from one parent distribution and testing these against observed populations. It follows that all known well-resolved stellar populations are consistent with having been drawn from the same parent mass distribution. The IMF UNIVERSALITY HYPOTHESIS therefore cannot be discarded despite the existence of the CLUSTER IMF THEOREM. This means that the currently existing star-formation theory fails to describe the stellar outcome, because it predicts a dependency of the IMF on the physical boundary conditions not observed. The IGIMF THEOREM, however, predicts a variation of galaxy-wide IMFs in dependence of the galaxy's star-formation rate even if the IMF UNIVERSALITY HYPOTHESIS is valid. This variation has now been observed in SDSS galaxy data. Detailed analysis of the binary properties in the very-low-mass star and brown dwarf (BD) mass regime on the one hand, and in the stellar regime on the other, shows there to be a discontinuity in the IMF near  $0.1 M_{\odot}$  such that BDs follow a separate distribution function. Very recent observations of the stellar population within 1 pc of the nucleus of the MW do suggest a top-heavy IMF, perhaps hinting at a variation of the star-formation outcome with tidal field and temperature thereby violating the IMF UNIVERSALITY HYPOTHESIS under these physically extreme conditions. Another violation of this hypothesis appears to emerge for extremely metal-poor stars such that the primordial IMF appears to have been depleted in low-mass stars.

### 1. Introduction

The stellar initial mass function (IMF),  $\xi(m) dm$ , where  $m$  is the stellar mass, is the parent distribution function of the masses of stars formed in *one* event. Here, the number of stars in the mass interval  $m, m + dm$  is  $dN = \xi(m) dm$ . The IMF is, strictly speaking, an abstract theoretical construct because any *observed* system of  $N$  stars merely constitutes a particular *representation* of this universal distribution function (Elmegreen 1997; Maíz Apellániz & Úbeda 2005). The probable existence of a *unique*  $\xi(m)$  can be inferred from observations of an ensemble of systems each consisting of  $N$  stars (e.g. Massey 2003). If, after corrections for (a) stellar evolution, (b) unknown multiple stellar systems, and (c) stellar-dynamical biases, the individual distributions of stellar masses are similar *within the expected statistical scatter*, then we (the community) deduce that the hypothesis that the stellar mass distributions are not the same can be excluded. That is, we make the case for a *universal, standard or canonical stellar*

*IMF* within the physical conditions probed by the relevant physical parameters (metallicity, density, mass) of the populations at hand.

A detailed related overview of the IMF can be found in Kroupa (2007a), and a review with an emphasis on the metal-rich problem is available in Kroupa (2007b), while Zinnecker & Yorke (2007) provide an in-depth review of the formation and distribution of massive stars. Elmegreen (2007) discusses the possibility that star-formation occurs in different modes with different IMFs.

## 2. The canonical or standard form of the stellar IMF

The *canonical stellar IMF* is a two-part-power law,  $\xi(m) \propto m^{-\alpha_i}$ , the only structure with confidence found so far being the change of index from the Salpeter/Massey value to a smaller one near  $0.5 M_\odot$ <sup>1</sup>:

$$\begin{aligned} \alpha_1 &= 1.3 \pm 0.3, & , & \quad 0.08 \lesssim m/M_\odot \lesssim 0.5, \\ \alpha_2 &= 2.3 \pm 0.5, & , & \quad 0.5 \lesssim m/M_\odot \lesssim 150. \end{aligned} \quad (1)$$

It has been corrected for bias through unresolved multiple stellar systems in the low-mass ( $m < 1 M_\odot$ ) regime using a multi-dimensional optimisation technique. The general outline of this technique is as follows (Kroupa, Tout & Gilmore 1993): first the correct form of the stellar-mass-luminosity relation is extracted using observed stellar binaries *and* theoretical constraints on the location, amplitude and shape of the minimum of its derivative near  $m = 0.3 M_\odot$ ,  $M_V \approx 12$ ,  $M_I \approx 9$  in combination with the observed shape of the nearby and deep Galactic-field LF. Having established the semi-empirical mass-luminosity relation of stars, which is an excellent fit to the most recent observational constraints by Delfosse et al. (2000), a model of the Galactic field is then calculated assuming a parametrised form for the MF and different values for the scale-height of the Galactic-disk, and different binary fractions in it. Measurement uncertainties and age and metallicity spreads must also be considered in the theoretical stellar population. Optimisation in this multi-parameter space (MF parameters, scale-height and binary population) against observational data leads to the canonical stellar MF for  $m < 1 M_\odot$ .

One important result from this work is the finding that the stellar luminosity function (LF) has a universal sharp peak near  $M_V \approx 12$ ,  $M_I \approx 9$ . It results from changes in the internal constitution of stars that drive a non-linearity in the stellar mass-luminosity relation.

A consistency-check is then performed as follows: the above MF is used in creating young populations of binary systems that are born in modest star clusters consisting of a few hundred stars. Their dissolution into the Galactic field is computed with an  $N$ -body code, and the resulting theoretical field is compared to the observed LFs (Kroupa 1995a,b). Further confirmation of the form of the canonical IMF comes from independent sources, most notably by Reid, Gizis & Hawley (2002) and also Chabrier (2003).

---

<sup>1</sup>The uncertainties in  $\alpha_i$  are estimated from the alpha-plot (§ 3.2.), as shown in fig. 5 in Kroupa (2002), to be about 95 % confidence limits

In the high-mass regime, Massey (2003) reports the same *slope or index*  $\alpha_3 = 2.3 \pm 0.1$  for  $m \gtrsim 10 M_\odot$  in many OB associations and star clusters in the Milky Way (MW), the Large- and Small-Magellanic clouds (LMC, SMC, respectively). It is therefore suggested to refer to  $\alpha_2 = \alpha_3 = 2.3$  as the *Salpeter/Massey slope or index*, given the pioneering work of Salpeter (1955) who derived this value for stars with masses  $0.4 - 10 M_\odot$ . However, multiplicity corrections await to be done once we learn more about how the components are distributed in massive stars (cf. Preibisch et al. 1999; Zinnecker 2003). Scalo (1986) found  $\alpha_{\text{MWdisk}} \approx 2.7$  ( $m \gtrsim 1 M_\odot$ ) from a very thorough analysis of OB star counts in the MW disk. Similarly, the star-count analysis of Reid et al. (2002) leads to  $2.5 \lesssim \alpha_{\text{MWdisk}} \lesssim 2.8$ , and Tinsley (1980), Kennicutt (1983; his “extended Miller-Scalo IMF”), Portinari et al. (2004) and Romano et al. (2005) find  $2.5 \lesssim \alpha_{\text{MWdisk}} \lesssim 2.7$ . That  $\alpha_{\text{MWdisk}} > \alpha_2$  naturally is shown in § 5.

The evidence for a universal upper mass cutoff near  $150 M_\odot$  (Weidner & Kroupa 2004; Figer 2005; Oey & Clarke 2005; Koen 2006; Maíz Apellániz et al. 2007) seems to be rather well established in populations with metallicities ranging from the LMC ( $Z \approx 0.008$ ) to the super-solar Galactic centre ( $Z \gtrsim 0.02$ ) such that the stellar mass function (MF) simply stops at that mass. This mass needs to be understood theoretically (see discussion in Kroupa & Weidner 2005 and Zinnecker & Yorke 2007).

Below the hydrogen-burning limit (see also § 4.) there is substantial evidence that the IMF flattens further to  $\alpha_0 \approx 0.3 \pm 0.5$  (Martín et al. 2000; Chabrier 2003; Moraux et al. 2004). Therefore, the canonical IMF most likely has a peak at  $0.08 M_\odot$ . Brown dwarfs, however, comprise only a few % of the mass of a population and are therefore dynamically irrelevant. The logarithmic form of the canonical IMF,  $\xi_L(m) = \ln(10) m \xi(m)$ , which gives the number of stars in  $\log_{10} m$ -intervals, also has a peak near  $0.08 M_\odot$ . However, the *system* IMF (of stellar single and multiple systems combined to system masses) has a maximum in the mass range  $0.4 - 0.6 M_\odot$  (Kroupa et al. 2003).

The above canonical or standard form has been derived from detailed considerations of star-counts thereby representing an *average* IMF: for low-mass stars it is a mixture of stellar populations spanning a large range of ages (0 – 10 Gyr) and metallicities ( $[\text{Fe}/\text{H}] \gtrsim -1$ ). For the massive stars it constitutes a mixture of different metallicities ( $[\text{Fe}/\text{H}] \gtrsim -1.5$ ) and star-forming conditions (OB associations to very dense star-burst clusters: R136 in the LMC). Therefore it can be taken as a canonical form, and the aim is to test the

IMF UNIVERSALITY HYPOTHESIS: the canonical IMF constitutes the parent distribution of all stellar populations.

Negation of this hypothesis would imply a variable IMF. Note that the work of Massey (2003) has already established the IMF to be invariable for  $m \gtrsim 10 M_\odot$  and for densities  $\rho \lesssim 10^5 \text{ stars/pc}^3$  and metallicity  $Z \gtrsim 0.002$ .

### 3. Universality of the IMF: resolved populations

The strongest test of the IMF UNIVERSALITY HYPOTHESIS (p. 3) is obtained by studying populations that can be resolved into individual stars. Since one also seeks co-eval populations at the same distance and with the same metallicity

to minimise uncertainties, star clusters and stellar associations would seem to be the test objects of choice. But before contemplating such work some lessons from stellar dynamics are useful:

### 3.1. Star clusters and associations

To access a pristine population one would consider observing star-clusters that are younger than a few Myr. However, such objects carry rather massive disadvantages: the pre-main sequence stellar evolution tracks are unreliable (Baraffe et al. 2002; Wuchterl & Tscharnuter 2003) such that the derived masses are uncertain by at least a factor of about two. Remaining gas and dust lead to patchy obscuration. Very young clusters evolve rapidly: the dynamical crossing time is  $t_{\text{cr}} = 2r_{\text{cl}}/\sigma_{\text{cl}}$ , where the cluster radii are typically  $r_{\text{cl}} < 1$  pc and for cluster masses  $M_{\text{cl}} > 10^3 M_{\odot}$  the velocity dispersion  $\sigma_{\text{cl}} > 2$  km/s such that  $t_{\text{cr}} < 1$  Myr. The inner regions of populous clusters have  $t_{\text{cr}} \approx 0.1$  Myr, and thus significant mixing and relaxation (relaxation time  $t_{\text{relax}} \approx t_{\text{cr}} 0.1 N/\ln N$ ) occurs there by the time the residual gas is expelled by the winds and photo-ionising radiation from massive stars, if they are present, being the case in clusters with  $N \gtrsim \text{few} \times 100$  stars. Much of this is summarised in Kroupa (2005). For example, the 0.5 – 2 Myr old Orion Nebula cluster (ONC), which is known to be super-virial with a virial mass about twice the observed mass (Hillenbrand & Hartmann 1998), has already expelled its residual gas and is expanding rapidly thereby probably having lost its outer stars (Kroupa, Aarseth & Hurley 2001). The super-virial state of young clusters makes measurements of their mass-to-light ratios a bad measure of the stellar mass within them (Bastian & Goodwin 2006; Goodwin & Bastian 2006), and rapid dynamical mass-segregation likewise makes “naive” measurements of the  $M/L$  ratios wrong (Boily et al. 2005; Fleck et al. 2006).

Massive stars ( $m > 8 M_{\odot}$ ) are either formed at the cluster centre or get there through dynamical mass segregation, i.e. energy equipartition (Bonnell, Larson & Zinnecker 2006). The latter process is *very rapid*, operating on a time-scale  $t_{\text{msgr}} \approx (m_{\text{av}}/m_{\text{massive}}) t_{\text{relax}}$ , where  $m_{\text{av}}, m_{\text{massive}}$  are the average and massive-star masses, respectively, and can occur within 1 Myr. A cluster core of massive stars is therefore either primordial or forms rapidly because of energy equipartition in the cluster, and it is dynamically highly unstable decaying within a few  $t_{\text{cr, core}}$ . The ONC, for example, should not be hosting a Trapezium as it is extremely volatile. The implication for the IMF is that the ONC and other similar clusters and the OB associations which stem from them must be very depleted in their massive-star content (Pflamm-Altenburg & Kroupa 2006).

Important for measuring the IMF are corrections for the typically high multiplicity fraction of the very young population. However, these are very uncertain because the binary population is in a state of change (fig.1 in Kroupa 2000).

The determination of an IMF relies on the assumption that all stars in a very young cluster formed together. Trapping of older field or OB association stars by the forming cluster has been found to be possible for ONC-type clusters (Pflamm-Altenburg & Kroupa 2007) and also for massive  $\omega$ -Cen-type clusters (Fellhauer, Kroupa & Evans 2006). Additionally, the sample of cluster stars may be contaminated by enhanced fore- and back-ground densities of field stars

due to focussing of stellar orbits during cluster formation (Pflamm-Altenburg & Kroupa 2007).

Thus, be it at the low-mass end or the high-mass end, the “IMF” estimated from very young clusters cannot be the true IMF. Statistical corrections for the above effects need to be applied and comprehensive  $N$ -body modelling is required.

Old open clusters in which most stars are on or near the main sequence are no better stellar samples: They are dynamically highly evolved, since they have left their previous concentrated and gas-rich state and so they contain only a fraction of the stars originally born in the cluster (Kroupa & Boily 2002; Weidner et al. 2007; Baumgardt & Kroupa 2007). The binary fraction is typically high and comparable to the Galactic field, but does depend on the initial density and the age of the cluster, the mass-ratio distribution of companions also. So, simple corrections cannot be applied equally for all old clusters. The massive stars have died, and secular evolution begins to affect the remaining stellar population (after gas expulsion) through energy equipartition. Baumgardt & Makino (2003) have quantified the changes of the MF for clusters of various masses and on different Galactic orbits. Near the half-mass radius the local MF resembles the global MF in the cluster, but the global MF becomes significantly depleted of its lesser stars already by about 20 % of the cluster’s disruption time.

Given that we are never likely to learn the exact dynamical history of a particular cluster, it follows that we can *never* ascertain the IMF for any individual cluster. This can be summarised concisely with the following theorem:

CLUSTER IMF THEOREM: The IMF cannot be extracted for any individual star cluster.

PROOF: For clusters younger than about 0.5 Myr star formation has not ceased and the IMF is therefore not assembled yet and the cluster cores consisting of massive stars have already dynamically ejected members (Pflamm-Altenburg & Kroupa 2006). For clusters with an age between 0.5 and a few Myr the expulsion of residual gas has lead to loss of stars (Kroupa et al. 2001). Older clusters are either still losing stars due to residual gas expulsion or are evolving secularly through evaporation driven by energy equipartition (Baumgardt & Makino 2003). There exists thus no time when all stars are assembled in an observationally accessible volume (i.e. a star cluster). End of proof.

Note that the CLUSTER IMF THEOREM implies that individual clusters cannot be used to make deductions on the similarity or not of their IMFs, unless a complete dynamical history of each cluster is available.

Notwithstanding this pessimistic theorem, it is nevertheless necessary to observe and study star clusters of any age. Combined with thorough and realistic  $N$ -body modelling the data do lead to essential *statistical* constraints on the IMF UNIVERSALITY HYPOTHESIS (p. 3). Such an approach is discussed in the next section.

### 3.2. The alpha plot

Scalo (1998) conveniently summarised a large part of the available observational constraints on the IMF of resolved stellar populations with the *alpha plot*, as used by Kroupa (2001, 2002) for explicit tests of the IMF UNIVERSALITY HY-

POTHESIS (p. 3) given the CLUSTER IMF THEOREM (p. 5). One example is presented in fig. 1 in Kroupa (2007b), which demonstrates that the observed scatter in  $\alpha(m)$  can be readily understood as being due to Poisson uncertainties (see also Elmegreen 1997, 1999) and dynamical effects, as well as arising from biases through unresolved multiple stars. Furthermore, there is no evident systematic change of  $\alpha$  at a given  $m$  with metallicity or density of the star-forming cloud. More exotic populations such as the Galactic bulge have also been found to have a low-mass MF indistinguishable from the canonical form (e.g. Zoccali et al. 2000). Thus the IMF UNIVERSALITY HYPOTHESIS cannot be falsified for known resolved stellar populations.

### 3.3. Very ancient resolved populations

Witnesses of the early formation phase of the MW are its globular clusters. Such  $10^{4-6} M_{\odot}$  clusters formed with individual star-formation rates of  $0.1 - 1 M_{\odot}/\text{yr}$  and densities  $\approx 5 \times 10^{3-5} M_{\odot}/\text{pc}^3$ . These are relatively high values, when compared with the current star-formation activity in the MW disk. For example, a  $5 \times 10^3 M_{\odot}$  Galactic cluster forming in 1 Myr corresponds to a star formation rate of  $0.005 M_{\odot}/\text{yr}$ . The alpha plot, however, does not support any significant systematic difference between the IMF of stars formed in globular clusters and present-day low-mass star formation. For massive stars, it can be argued that the mass in stars more massive than  $8 M_{\odot}$  cannot have been larger than about half the cluster mass, because otherwise the globular clusters would not be as compact as they are today. This constrains the IMF to have been close to the canonical IMF (Kroupa 2001).

A particularly exotic star-formation mode is thought to have occurred in dwarf-spheroidal (dSph) satellite galaxies. The MW has about 19 such satellites at distances from 50 to 250 kpc (Metz & Kroupa 2007). These objects have stellar masses and ages comparable to those of globular clusters but are 10 – 100 times larger and are thought to have 10 – 100 times more mass in dark matter than in stars. They also show evidence for complex star-formation activity and metal-enrichment histories and must have therefore formed under rather exotic conditions. Nevertheless, the MFs in two of these satellites are found to be indistinguishable from those of globular clusters in the mass range  $0.5 - 0.9 M_{\odot}$ , thus again showing consistency with the canonical IMF (Grillmair et al. 1998; Feltzing, Gilmore & Wyse 1999).

### 3.4. The Galactic bulge and centre

For low-mass stars the Galactic bulge has been shown to have a MF indistinguishable from the canonical form (Zoccali et al. 2000). However, abundance patterns of bulge stars suggest the IMF to have been top heavy (Ballero et al. 2007), which may be a result of extreme star-burst conditions valid in the formation of the bulge (Zoccali et al. 2006).

Even closer to the Galactic centre, Hertzsprung-Russell-diagram modelling of the stellar population within 1 pc of Sgr A\* suggests the IMF to have always been top-heavy there (Maness et al. 2007). Perhaps this is the long-sought after evidence for a variation of the IMF under very extreme conditions, in this case a strong tidal field and higher temperatures.

### 3.5. Population III: the primordial IMF

Most theoretical workers agree that the primordial IMF ought to be top heavy because the ambient temperatures were much higher and the lack of metals did not allow gas clouds to cool and to fragment into sufficiently small cores. The existence of extremely metal-poor low-mass stars with chemical peculiarities is interpreted to mean that low-mass stars could form under extremely metal-poor conditions, but that their formation was suppressed in comparison to later star-formation (Tumlinson 2007). Modelling of the formation of stellar populations during cosmological structure formation suggests that low-mass population III stars should be found within the Galactic halo *if* they formed. Their absence to-date would imply a primordial IMF depleted in low-mass stars (Brook et al. 2007).

## 4. Very low-mass stars (VLMSs) and brown dwarfs (BDs)

The origin of BDs and some VLMSs is being debated fiercely. One camp believes these objects to form as stars do, because the star-formation process does not know where the hydrogen burning mass limit is (e.g. Eislöffel & Steinacker 2007). The other camp believes that BDs cannot form exactly like stars through continued accretion because the conditions required for this to occur in molecular clouds are far too rare (e.g. Reipurth & Clarke 2001; Goodwin & Whitworth 2007).

If BDs and VLMSs form like stars then they should follow the same pairing rules. In particular, BDs and G dwarfs would pair in the same manner as M dwarfs and G dwarfs. Kroupa et al. (2003) tested this hypothesis by constructing  $N$ -body models of Taurus-Auriga-like groups and Orion-Nebula-like clusters finding that it leads to far too many star-BD and BD-BD binaries with the wrong semi-major axis distribution. Instead, star-BD binaries are rare while BD-BD binaries have a semi-major axis distribution significantly narrower than that of star-star binaries. The hypothesis of a star-like origin of BDs must therefore be discarded. BDs and some VLMSs form a separate population, which is however linked to that of the stars.

Thies & Kroupa (2007) re-address this problem with a detailed analysis of the underlying MF of stars and BDs given observed MFs of four populations, Taurus, Trapezium, IC348 and the Pleiades. By correcting for unresolved binaries in all four populations, by taking into account the different pairing rules of stellar and VLMS and BD binaries, a significant discontinuity of the MF emerges. BDs and VLMSs therefore form a truly separate population from that of the stars and can be described by a single power-law MF,  $\xi_{\text{BD}}$ , with index  $\alpha_0 \approx 0.3$  and  $\xi_{\text{BD}}(0.075 M_{\odot}) = (0.2 - 0.3) \xi(0.075 M_{\odot})$ , where  $\xi$  is the canonical stellar IMF. This implies that about one BD forms per 5 stars in all four populations.

This strong correlation between the number of stars and BDs, and the similarity of the BD MF in the four populations implies that the formation of BDs is closely related to the formation of stars. Indeed, the truncation of the binary binding energy distribution of BDs at a high energy suggests that energetic processes must be operating in the production of BDs, as discussed in Thies & Kroupa (2007).

## 5. Composite populations: the IGIMF

The vast majority of all stars form in embedded clusters and so the correct way to proceed to calculate a galaxy-wide stellar IMF is to add-up all the IMFs of all star-clusters born in one “star-formation epoch”. Such “epochs” may be identified with the Zoccali et al. (2006) star-burst events creating the Galactic bulge. In disk galaxies they may be related to the time-scale of transforming the inter-stellar matter to star clusters along spiral arms. Addition of the clusters born in one epoch gives the *integrated galactic initial mass function*, the IGIMF (Kroupa & Weidner 2003).

IGIMF DEFINITION: The IGIMF is the IMF of a composite population which is the integral over a complete ensemble of simple stellar populations.

Note that a *simple population* has a mono-metallicity and a mono-age distribution and is therefore a star cluster. Age and metallicity distributions emerge for massive populations with  $M_{\text{cl}} \gtrsim 10^6 M_{\odot}$  (e.g.  $\omega$  Cen) indicating that such objects, which also have relaxation times comparable to or longer than a Hubble time, are not “simple”. A *complete ensemble* is a statistically complete representation of the initial cluster mass function (ICMF) in the sense that the actual mass function of  $N_{\text{cl}}$  clusters lies within the expected statistical variation of the ICMF.

IGIMF THEOREM: The IGIMF is steeper than the canonical IMF if the IMF UNIVERSALITY HYPOTHESIS holds.

PROOF: Weidner & Kroupa (2006) calculate that the IGIMF is steeper than the canonical IMF for  $m \gtrsim 1 M_{\odot}$  if the IMF UNIVERSALITY HYPOTHESIS holds. The steepening becomes negligible if the power-law ICMF is flatter than  $\beta \approx 1.8$ . End of proof.

It may be argued that IGIMF=IMF (e.g. Elmegreen 2006) because when a star cluster is born, its stars are randomly sampled from the IMF up to the most massive star possible. On the other hand, the physically-motivated ansatz by Weidner & Kroupa (2005, 2006) of taking the mass of a cluster as the constraint and of including the observed correlation between the maximal star mass and the cluster mass, yields an IGIMF which is equal to the canonical IMF for  $m \lesssim 1.5 M_{\odot}$  but which is systematically steeper above this mass. By incorporating the observed maximal-cluster-mass vs star-formation rate of galaxies for the youngest clusters (Weidner, Kroupa & Larsen 2004), it follows for  $m \gtrsim 1.5 M_{\odot}$  that low-surface-brightness (LSB) galaxies have very steep IGIMFs while normal or  $L_*$  galaxies have Scalo-type IGIMFs, i.e.  $\alpha_{\text{IGIMF}} = \alpha_{\text{MWdisk}} > \alpha_2$  (§ 2.) follows naturally. This systematic shift of  $\alpha_{\text{IGIMF}}$  ( $m \gtrsim 1.5 M_{\odot}$ ) with galaxy type implies that less-massive galaxies have a significantly suppressed supernova II rate per low-mass star. They also show a slower chemical enrichment such that the observed metallicity–galaxy-mass relation can be nicely accounted for (Koeppen, Weidner & Kroupa 2007).

Strikingly, the IGIMF variation has now been directly measured by Hoyersten & Glazebrook (2007) using galaxies in the SDSS. Lee et al. (2004) have indeed found LSBs to have bottom-heavy IMFs, while Portinari et al. (2004)



and Romano et al. (2005) find the MW disk to have a steeper-than Salpeter IMF for massive stars which is, in comparison with Lee et al., much flatter than the IMF of LSBs, as required by the IGIMF THEOREM.

## 6. Origin of the IMF: theory vs observations

General physical concepts such as coalescence of proto-stellar cores, mass-dependent focussing of gas accretion onto proto-stars, stellar feedback, and fragmentation of molecular clouds lead to predictions of systematic variations of the IMF with changes of the physical conditions of star formation (Murray & Lin 1996; Elmegreen 2004; Tilley & Pudritz 2005; but see Casuso & Beckman 2007 for a simple cloud coagulation/dispersal model leading to an invariant mass distribution). Thus, the thermal Jeans mass of a molecular cloud decreases with temperature and increasing density, implying that for higher metallicity (= stronger cooling) and density the IMF should shift on average to smaller stellar masses (e.g. Larson 1998; Bonnell, Larson & Zinnecker 2006). The entirely different notion that stars regulate their own masses through a balance between feedback and accretion also implies smaller stellar masses for higher metallicity due in part to more dust and thus more efficient radiation pressure (Adams & Fatuzzo 1996; Adams & Laughlin 1996)

Klessen, Spaans & Jappsen (2007) report state-of-the art calculations of star-formation under physical conditions as found in molecular clouds near the Sun and they are able to reproduce the canonical IMF. Applying the same computational technology to the conditions near the Galactic centre they obtain a theoretical IMF in agreement with the previously reported apparent decline of the stellar MF in the Arches cluster below about  $6 M_{\odot}$ . Kim et al. (2006) published their observations of the Arches cluster on the astro-physics preprint archive shortly after Klessen et al. (2007) and performed the necessary state-of-the art  $N$ -body calculations of the dynamical evolution of this young cluster, revising our knowledge significantly. In contradiction to Klessen et al. (2007) they find that the MF continues to increase down to their 50 % completeness limit ( $1.3 M_{\odot}$ ) with a power-law exponent only slightly shallower than the canonical Massey/Salpeter value once mass-segregation is corrected for. This situation is demonstrated in Fig. 1.

Observations of cloud cores appear to suggest that the canonical IMF is frozen-in already at the pre-stellar cloud-core level (Motte et al. 1998; 2001). Nutter & Ward-Thompson (2007) and Alves, Lombardi & Lada (2007) find, however, the pre-stellar cloud cores to be distributed according to the same shape as the canonical IMF, but shifted to larger masses by a factor of about three or more. This is taken to perhaps mean a star-formation efficiency per star of 30 % or less independently of stellar mass. The interpretation of such observations in view of multiple star formation in each cloud-core is being studied by Goodwin et al. (2007), while Krumholz (2007, these proceedings) outlines current theoretical understanding of how massive stars form out of the massive pre-stellar cores.

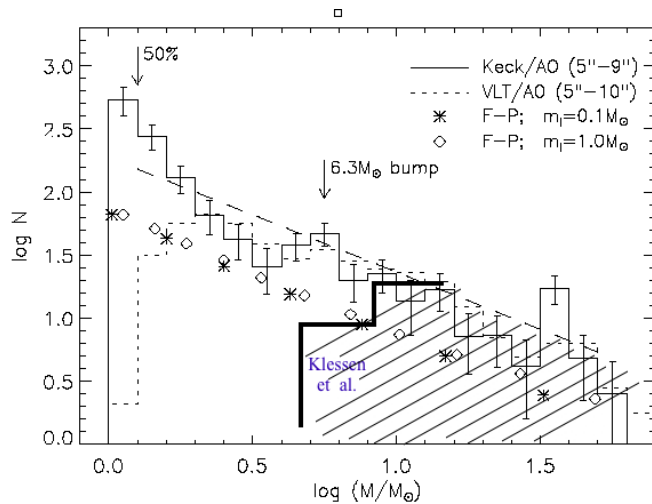


Figure 1. The observed mass function of the Arches cluster near the Galactic centre by Kim et al. (2006) shown as the thin histogram is confronted with the theoretical MF for this object calculated with the SPH technique by Klessen et al. (2007). The latter has a down-turn incompatible to the observations therewith ruling out a theoretical understanding of the stellar mass spectrum (“one counter-example suffices to bring-down a theory”). One possible reason for the theoretical failure may be the assumed turbulence driving. For details on the figure see Kim et al. (2006).

## 7. Conclusions

The IMF UNIVERSALITY HYPOTHESIS, the CLUSTER IMF THEOREM and the IGIMF THEOREM have been stated. Furthermore,

1. The stellar luminosity function has a pronounced maximum at  $M_V \approx 12$ ,  $M_I \approx 9$  which is universal and well understood as a result of stellar physics. Thus by counting stars on the sky we can look into their interiors.
2. Unresolved multiple systems must be accounted for when the MFs of different stellar populations are compared.
3. BDs and some VLMSs form a separate population which correlates with the stellar content; there is a discontinuity in the MF near the star/BD mass transition.
4. The canonical IMF (eq. 1) fits the solar-neighbourhood star counts and *all* resolved stellar populations available to-date. Recent data near the Galactic centre suggest a top-heavy IMF, perhaps hinting at a possible variation with conditions, although the results on the Arches cluster near the Galactic centre are not entirely supportive of this statement.
5. *Simple* stellar populations are found in individual star clusters with  $M_{cl} \lesssim 10^6 M_\odot$ . These have the canonical IMF.

6. *Composite* populations describe entire galaxies. They are a result of many epochs of star-cluster formation and are described by the IGIMF THEOREM.
7. The IGIMF above  $\approx 1 M_{\odot}$  is steep for LSB galaxies, flattening to the Scalo slope ( $\alpha_{\text{IGIMF}} \approx 2.7$ ) for  $L_*$  disk galaxies. This is nicely consistent with the IMF UNIVERSALITY HYPOTHESIS in the context of the IGIMF THEOREM.
8. Therefore, the IMF UNIVERSALITY HYPOTHESIS can not be excluded despite the CLUSTER IMF THEOREM for conditions  $\rho \lesssim 10^5$  stars/pc<sup>3</sup>,  $Z \gtrsim 0.002$  and non-extreme tidal fields.
9. Modern star-formation computations appear to give wrong results concerning the shape of the stellar IMF.
10. The stellar IMF appears to be frozen-in at the pre-stellar cloud-core stage therewith probably being a result of the processes leading to the formation of self-gravitating molecular clouds.

**Acknowledgments.** I am very grateful to the organisers for a most enjoyable meeting. I also thank Thomas Maschberger and Jan Pflamm-Altenburg for helpful discussions and comments, and my collaborators for important input. Parts of this work were supported by the DFG.

## References

- Adams F. C., & Fatuzzo M. 1996. *ApJ*, 464, 256  
Adams F. C., & Laughlin G. 1996. *ApJ*, 468, 586  
Alves, J., Lombardi, M., & Lada, C. J. (2007). *A&A*, 462, L17  
Ballerio, S., Kroupa, P., Matteucci, F. 2007. *MNRAS*, 467, 117  
Baraffe, I., Chabrier, G., Allard, F., & Hauschildt, P. H. 2002. *A&A*, 382, 563  
Baumgardt, H., & Kroupa, P., 2007. *MNRAS*, in press, astro-ph/0707.1944  
Baumgardt H., Makino J. 2003. *MNRAS*, 340, 227  
Bastian, N., & Goodwin, S. P. 2006. *MNRAS*, 369, L9  
Boily, C. M., Lançon, A., Deiters, S., & Heggie, D. C. 2005. *ApJ*, 620, L27  
Bonnell, I. A., Larson, R., & Zinnecker, H. 2006. *Proto Stars and Planets V*, astro-ph/0603447  
Brook, C. B., Kawata, D., Scannapieco, E., Martel, H., & Gibson, B. K. 2007. *ApJ*, 661, 10  
Casuso, E., & Beckman, J.E., 2007. *ApJ*, 656, 897  
Chabrier G. 2003. *PASP*, 115, 763  
Delfosse, X., Forveille, T., Ségransan, D., Beuzit, J.-L., Udry, S., Perrier, C., & Mayor, M. 2000. *A&A*, 364, 217  
Eisloffel, J., & Steinacker, J. (2007). in *The Formation of Low-Mass-Protostars and Proto-Brown Dwarfs* at the 14th Cambridge Workshop on Cool Stars, Stellar Systems and the Sun in Pasadena, 6 – 10 November 2006, astro-ph/0701525  
Elmegreen B. G. 1997. *ApJ*, 486, 944  
Elmegreen B. G. 1999. *ApJ*, 515, 323  
Elmegreen, B. G. 2004. *MNRAS*, 354, 367  
Elmegreen, B. G. 2006. *ApJ*, 648, 572  
Elmegreen, B. G. 2007. *ApJ*, in press, astro-ph/0707.2252  
Fellhauer, M., Kroupa, P., & Evans, N. W. 2006. *MNRAS*, 372, 338

- Feltzing, S., Gilmore, G., & Wyse, R. F. G. 1999. *ApJ*, 516, L17
- Figer D. F. 2005. *Nature*, 434, 192
- Fleck, J.-J., Boily, C. M., Lançon, A., & Deiters, S. 2006. *MNRAS*, 369, 1392
- Goodwin, S. P., & Bastian, N. 2006. *MNRAS*, 373, 752
- Goodwin, S. P., & Whitworth, A. 2007. *A&A*, 466, 943
- Goodwin, A. P., Nutter, D., Kroupa, P., Ward-Thompson, D., & Whitworth, A.P., 2007. in preparation
- Grillmair, C. J., et al. 1998. *AJ*, 115, 144
- Hillenbrand, L. A., & Hartmann, L. W. 1998. *ApJ*, 492, 540
- Hoversten, E. A. & Glazebrook, K. 2007. *ApJ*, in press
- Kennicutt, R. C., Jr. 1983. *ApJ*, 272, 54
- Kim, S. S., Figer, D. F., Kudritzki, R. P., & Najarro, F. 2006. *ApJL*, 653, L113
- Klessen, R. S., Spaans, M., & Jappsen, A.-K. 2007. *MNRAS*, 374, L29
- Koen, C. 2006. *MNRAS*, 365, 590
- Koeppen, J., Weidner, C., & Kroupa, P. 2007. *MNRAS* 375, 673
- Kroupa P., 1995a. *ApJ*, 453, 350
- Kroupa P. 1995b. *ApJ*, 453, 358
- Kroupa, P. 2000. *New Astronomy*, 4, 615
- Kroupa P. 2001. *MNRAS*, 322, 231
- Kroupa P. 2002. *Science*, 295, 82
- Kroupa, P. 2005. in *ESA SP-576: The Three-Dimensional Universe with Gaia*, 629 (astro-ph/0412069)
- Kroupa, P. 2007a. in *Resolved Stellar Populations*, Cancun, Mexico, 18-22 April 2005, eds. D. Valls-Gabaud & M. Chavez, *ASP Conf. Ser.* (in press), astro-ph/0703124
- Kroupa, P. 2007b. in *The Metal Rich Universe*, La Palma, June 2006, eds. G. Israelian & G. Meynet, Cambridge University Press, astro-ph/0703282
- Kroupa, P., Boily, C. M. 2002. *MNRAS*, 336, 1188
- Kroupa P., & Weidner C. 2003. *ApJ*, 598, 1076
- Kroupa, P., & Weidner, C. 2005. *IAU Symposium*, 227, 423, astro-ph/0507582
- Kroupa P., Aarseth S., & Hurley J. 2001. *MNRAS*, 321, 699
- Kroupa, P., Tout, C. A., & Gilmore, G. 1993. *MNRAS*, 262, 545
- Kroupa, P., Bouvier, J., Duchêne, G., & Moraux, E. 2003. *MNRAS*, 346, 354
- Krumholz, M. R. 2007. in *Pathways Through an Eclectic Universe*, eds. J. Knapen, T. Mahoney, & A. Vazdekis, astro-ph/0706.3702
- Larson, R. B. 1998. *MNRAS*, 301, 569
- Lee, H.-c., Gibson, B.K., Flynn, C., Kawata, D., & Beasley, M.A. 2004. *MNRAS*, 353, 113
- Maíz Apellániz, J., & Úbeda, L. 2005. *ApJ*, 629, 873
- Maíz Apellániz, J., Walborn, N. R., Morrell, N. I., Niemela, V. S., & Nelan, E. P. 2007. *ApJ*, 660, 1480
- Maness, H., Martins, F., Trippe, S., et al. 2007. *ApJ*, in press, astro-ph/0707.2382
- Martín, E. L., Brandner, W., Bouvier, J., Luhman, K. L., Stauffer, J., Basri, G., Zapatero Osorio, M. R., & Barrado y Navascués, D. 2000. *ApJ*, 543, 299
- Massey P. 2003. *AR&A*, 41, 15
- Metz, M., & Kroupa, P. 2007. *MNRAS*, 376, 387
- Moraux, E., Bouvier, J., & Clarke, C. 2004. in *SF2A-2004: Semaine de l'Astrophysique Française*, ed. F. Combes, D. Barret, T. Contini, F. Meynadier and L. Pagani, 251
- Motte, F., Andre, P., & Neri, R. 1998. *A&A*, 336, 150
- Motte, F., André, P., Ward-Thompson, D., & Bontemps, S. 2001. *A&A*, 372, L41
- Murray, S. D., & Lin, D. N. C. 1996. *ApJ*, 467, 728
- Nutter, D., & Ward-Thompson, D. 2007. *MNRAS*, 374, 1413
- Oey, M. S., & Clarke, C. J. 2005. *ApJL*, 620, L43
- Pflamm-Altenburg, J., & Kroupa, P. 2006. *MNRAS*, 373, 295
- Pflamm-Altenburg, J., & Kroupa, P. 2007. *MNRAS*, 375, 855

- Portinari, L., Sommer-Larsen, J., & Tantalo, R. 2004. MNRAS, 347, 691
- Preibisch T., Balega Y., Hofmann K., Weigelt G., & Zinnecker H. 1999. New Astronomy, 4, 531
- Reid I. N., Gizis J. E., & Hawley S. L. 2002. AJ, 124, 2721
- Reipurth, B., & Clarke, C. 2001. AJ, 122, 432
- Romano, D., Chiappini, C., Matteucci, F., & Tosi, M. 2005. A&A, 430, 491
- Salpeter E. E. 1955. ApJ, 121, 161
- Scalo J. M. 1986. Fundamentals of Cosmic Physics, 11, 1
- Scalo J. 1998. in ASP Conf. Ser. 142: The Stellar Initial Mass Function (38th Hermonieux Conference) The IMF Revisited: A Case for Variations. pp 201
- Thies, I., & Kroupa, P. 2007. ApJ, in press
- Tilley, D. A., Pudritz, R. E. 2005. Protostars and Planets, V, 8473
- Tinsley, B. M. 1980. Fundamentals of Cosmic Physics, 5, 287
- Tumlinson, J. 2007. ApJ, in press, astro-ph/0707.0816
- Weidner C., & Kroupa P. 2004. MNRAS, 348, 187
- Weidner C., & Kroupa P. 2005. ApJ, 625, 754
- Weidner, C., & Kroupa, P. 2006. MNRAS, 365, 1333
- Weidner C., Kroupa P., & Larsen S. S. 2004. MNRAS, 350, 1503
- Weidner, C., Kroupa, P., Nürnberger, D., & Sterzik, M., 2007. MNRAS, 376, 1879
- Wuchterl, G., & Tscharnuter, W. M. 2003. A&A, 398, 1081
- Zinnecker, H. 2003. IAU Symposium, 212, 80
- Zinnecker, H., & Yorke, H. W., 2007. astro-ph/0707.1279
- Zoccali, M., Cassisi, S., Frogel, J. A., Gould, A., Ortolani, S., Renzini, A., Rich, R. M., & Stephens, A. W. 2000. ApJ, 530, 418
- Zoccali, M., et al. 2006. A&A, 457, L1